The force

● The force :

The force is defined as the effect of a natural body

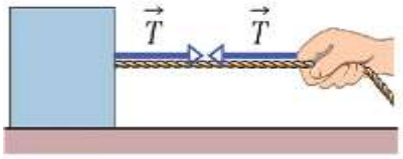
Upon another one .

● Kinds of forces :

[1] Tension force (T)

As the force in the string (or the rope)

When carrying a body at it .



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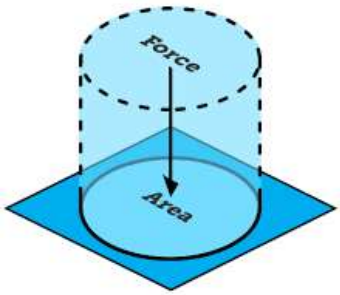
[2] Pressure force (P)

As the force that appears when abody

Stabilized on a surface



Pressure

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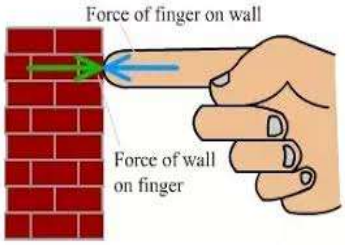
[3] Reaction force (r)

As the reaction of a smooth surface

On a body stabilized on it .



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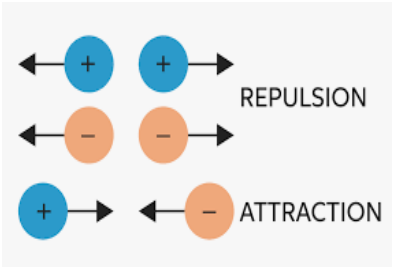
[4] Attraction forces and repulsion force

As the force which formed between

Magnetic poles , electric charges .



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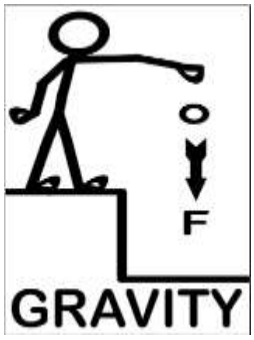
[5] Gravitational force (weights)

If we let a body in the air , then it will drop

down towards the earth .

→ Notice that : $W = M \times G$

The weight = the body mass × accelertion of gravity



● Determination of the force

→ the force is determined by :

- The magnitude of the force .
- The direction of the force .
- The point of action of the force .

● the measurement units of the force

The magnitude of the force is measured by weight units .

Where : 1 kg.wt = 10^3 gm.wt

1 newton = 10^5 dyne

1 kg.wt = 9.8 newton = 980 dyne

● the force can be expressed as :

- $\vec{f}$ = (x , y) → the cartesian form
- $\vec{f} = x\,\vec{i} + y\,\vec{j}$ → in term of the fundamental unit vectors
- $\vec{f}$ = ($\|\vec{f}\|$, θ) → the polar form

Resultant of two forces Meeting at a point

if :

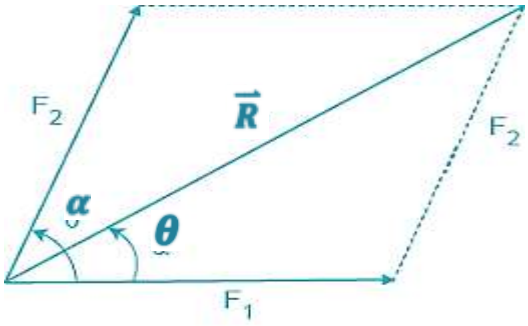
$\overrightarrow{f_1}$, $\overrightarrow{f_2}$ two forces meet at a point O

And : $\overrightarrow{OC}$ represents the resultant $\vec{R}$

Then :

→ ∠ α between $\overrightarrow{f_1}$, $\overrightarrow{f_2}$

→ ∠ θ between $\vec{R}$, $\overrightarrow{f_1}$



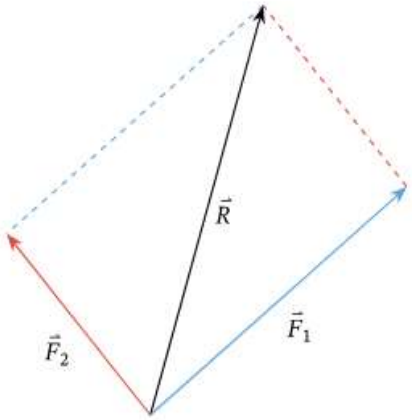
SO :

→ R = $\sqrt{(f_1)^2 + (f_2)^2 + 2 f_1 f_2 \cos \alpha}$ (magnitude)

→ $\tan \theta = \frac{f_2 \sin \alpha}{f_1 + f_2 \cos \alpha}$ (direction)

Special Cases :

1) if the two forces are perpendiular



→ $\alpha = 90°$ where : $\sin\alpha = 1$, $\cos\alpha = 0$

Then :

- $R = \sqrt{(f_1)^2 + (f_2)^2 + 2 f_1 f_2 \cos 90}$ → $R = \sqrt{(f_1)^2 + (f_2)^2}$

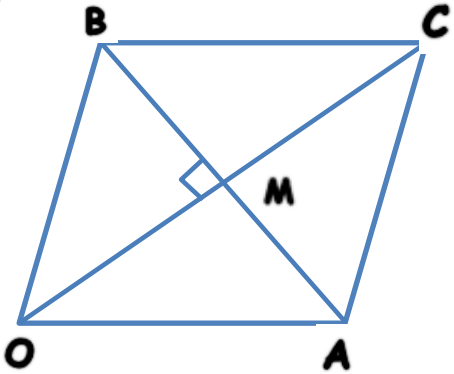
- $\tan\theta = \frac{f_2 \sin 90}{f_1 + f_2 \cos 90}$ → $\tan\theta = \frac{f_2}{f_1}$

---

1) if the two forces are equal in magnitude ($f_1 = f_2 = f$)

Where : $\vec{R}$ bisects the angle between the two forcse , $\theta = \frac{\alpha}{2}$

(the parallelogram exchanges to a rhombus)



then: $R = OC = 2\,OM = 2\,OA \cos\frac{\alpha}{2}$

i.e. $R = 2 f \cos\frac{\alpha}{2}$

Notics that : if : $\alpha = 120$ → SO : R = F

---

1) if the two forces have the same line of action

And the same direction

→ $\alpha = 0^\circ$, $\cos \alpha = 1$

→ $R = \sqrt{(f_1)^2 + (f_2)^2 + 2 f_1 f_2 \cos 0}$ → $R = \sqrt{(f_1 + f_2)^2}$

→ $R = f_1 + f_2$ (maximum value)

---

1) if the two forces have the same line of action

And in the opposite direction

→ $\alpha = 180^\circ$, $\cos \alpha = -1$

→ $R = \sqrt{(f_1)^2 + (f_2)^2 + 2 f_1 f_2 \cos 180}$ → $R = \sqrt{(f_1 - f_2)^2}$

→ $R = |f_1 - f_2|$ (minimum value)

→ the resultant have the direction of the greater force .

---

1) if the two forces are egual in magnitude and have the same line of action And in the opposite direction

→ $f_1 = f_2 = f$

→ $\alpha = 180°$

→ R = zero

---

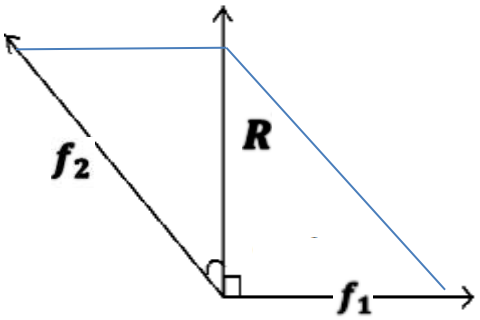
1) if the resultant is perpendicilar to the first force

→ $\theta = 90°$

→ $R^2 = (f_2)^2 - (f_1)^2$ (Pythagoras)

→ $f_1 + f_2 \cos\alpha = 0$

→ $\cos\alpha = \frac{-f_1}{f_2}$, $f_1 < f_2$



→ the resultant is perpendicular to the smaller force

→ two forces of magnitude 5 n , 3 n act at apoint and include an angle of measure 60° , find their resultant in magnitude and direction ?

SOL

→ $R = \sqrt{(f_1)^2 + (f_2)^2 + 2 f_1 f_2 \cos \alpha}$

$= \sqrt{(5)^2 + (3)^2 + 2 \times 5 \times 3 \times \cos 60} = n$

→ $\tan \theta = \frac{f_2 \sin \alpha}{f_1 + f_2 \cos \alpha} = \frac{3 \sin 60}{5 + 3 \cos 60}$

∴ $\theta =$

→ two forces act at apoint . the greatest value of their resultant is 32 n , and the smallest value is 12 n

find : the magnitude of each of them , then find the magnitude of their resultant if the measure of the included angle between them is 60° ?

SOL

$f_1 + f_2 = 32$, $f_1 - f_2 = 12$

$\therefore 2 f_1 = 44$ $\therefore f_1 = 22$ n $\therefore f_2 = 12$ n

→ $R = \sqrt{(f_1)^2 + (f_2)^2 + 2 f_1 f_2 \cos \alpha}$

$= \sqrt{(22)^2 + (12)^2 + 2 \times 22 \times 12 \times \cos 60} =$ n

---

▶ Note >>>

If : the greatest value is A and the smallest value is B

Then : $R \in [A , B]$

→ two forces of magnitude 4f n , 3f n act at a point

Find the measure of the angle between them

if the magnitude of the resultant is $\sqrt{13}$ f ?

SOL

→ $R^2 = (f_1)^2 + (f_2)^2 + 2 f_1 f_2 \cos \alpha$

∴ $13 f^2 = 16 f^2 + 9 f^2 + 2 \times 4 f \times 3 f \times \cos \alpha$

∴ $13 f^2 = 25 f^2 + 24 f^2 \times \cos \alpha$

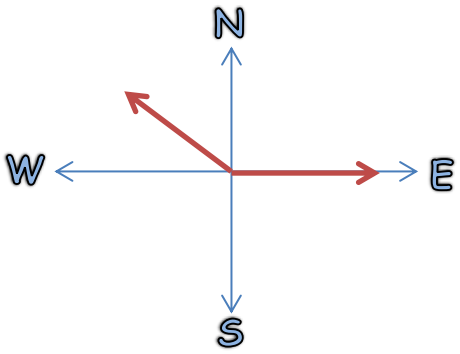
∴ $\cos \alpha = \frac{13}{49}$ $\Longrightarrow$ $\alpha =$

→ two forces of magnitude 5 n , $5\sqrt{2}$ n act at a point

The first towards East , the secound is towards Western North . prove that the magnitude of the resultant = the magnitude of the first force , and find the measure of the angle which the resultant makes with each of the two forces

SOL

$\alpha = 90 + 45 = 135$



$$R = \sqrt{(f_1)^2 + (f_2)^2 + 2 f_1 f_2 \cos \alpha}$$

$$= \sqrt{(5)^2 + (5\sqrt{2})^2 + 2 \times 5 \times 5\sqrt{2} \times \cos 135} = \quad n$$

→ the magnitude of the resultant of two perpendicular forces is 50 n . if the resultant makes with the first force an angle of measure 30° , find the magnitude of each of these two forces

SOL

$\because \tan\theta = \frac{f_2}{f_1}$

$\therefore \tan 30° = \frac{f_2}{f_1}$ $\therefore \frac{f_2}{f_1} = \frac{1}{\sqrt{3}}$

$\therefore f_1 = \sqrt{3}\, f$, $f_2 = f$

$\therefore R^2 = (f_1)^2 + (f_2)^2$

$\therefore (50)^2 = 3f^2 + f^2$ $\therefore f = \sqrt{\frac{(50)^2}{4}} = 25$ n

⟹ the resultant of two fores 6 newton and 8 newten

Could be ..... newten

A 20 B 15 C 12 D 1

SOL

→ R ∈ [2 , 14] → R could be 12 n

⟹ Two forces act at a point . the magnitude of the forces Are 6 , 3 newten , and their resultant is perpendicular to one of them , then the magnitude of their resultant = ... n

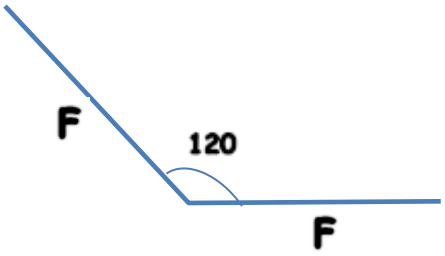
A 3 B $3\sqrt{3}$ C 6 D $6\sqrt{3}$

SOL

→ R = $\sqrt{6^2 - 3^2}$ = $3\sqrt{3}$

⟹ in the opposite figure :

R = ...



A 2F B F C $\sqrt{3}$ F D zero

SOL

→ R = 2F cos 60 = f

⟹ Two forces of magnitude 3 , F newten , and the measure of the angle between them is 120 , if their resultant is perpendicular to the first force , so the value of F is ..... n

A 1.5 B 3 C $3\sqrt{3}$ D 6

SOL

→ $\cos 120^\circ = \frac{-3}{f}$ → f = 6 n

⟹ Two forces of equals magnitude , enclosing between them an angle of measure $\frac{\pi}{2}$, if the magnitude of their resultant is 8 n , then the value of eash force measured in newton is .....

A $2\sqrt{2}$ B 4 C $4\sqrt{2}$ D 8

SOL

→ $8 = 2F \cos 45$ → $f = 4\sqrt{2}$

Or → $8 = \sqrt{2f^2}$ → $f = 4\sqrt{2}$

⟹ Two forces of magnitude 6 N and 8 N , if the magnitude of their resultant is 2 N , then the measure of the angle between the two forces is .....

A 30° B 90° C 180° D 270°

SOL

→ $4 = 36 + 64 + 2(6)(8)\cos\alpha$

→ $\cos\alpha = \frac{4-36-64}{2(6)(8)}$ → $\alpha = 180^\circ$

⟹ Two forces of magnitude 12 N and 15 N act at a particle and the measure of the enclosing angle between them is θ° , where $\cos\theta = \frac{-4}{5}$, then the measure of the included angle between the resultant and the first force = ... °

A zero B 30 C 90 D 36° 52'

SOL

$\therefore \cos\theta = \frac{-f_1}{f_2}$

∴ the angle between the resultant and the first force = 90°

⟹ Two forces act at a point . the magnitudes of the two forces are 5 , 3 newten , then the magnitude of their resultant measure by newten ∈ .....

A [2 , 8] B] 2 , 8 [C [3 , 5] D] 3 , 5 [

SOL

the greatest value = 5 + 3 = 8 , the smallest value = 5 - 3 = 2

Then : R ∈ [2 , 8]

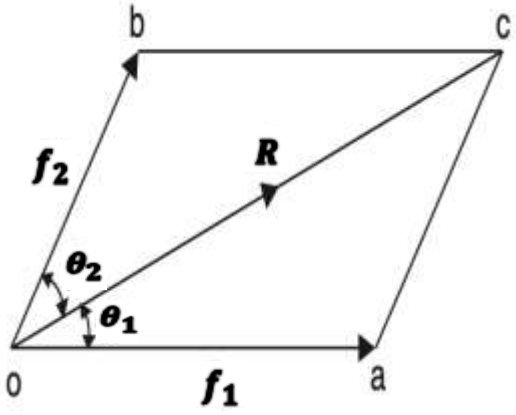
Forces resolution into two components

• Resolution of a known force into two known directions :

▶ $\frac{f_1}{\sin\theta_2} = \frac{f_2}{\sin\theta_1} = \frac{R}{\sin(\theta_1+\theta_2)}$

▶ $f_1 = \frac{R\sin\theta_2}{\sin(\theta_1+\theta_2)}$

▶ $f_2 = \frac{R\sin\theta_1}{\sin(\theta_1+\theta_2)}$

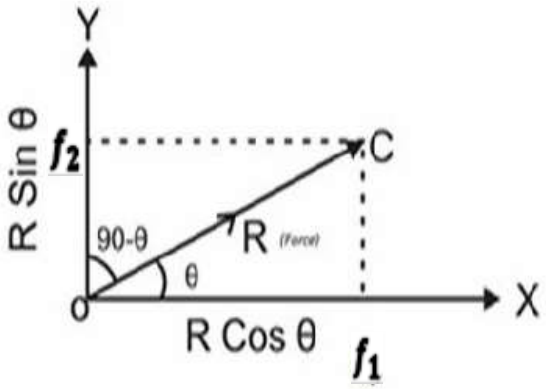


• Resolution of the force into two perpendicular directions :

▶ $\frac{f_1}{\sin(90-\theta)} = \frac{f_2}{\sin\theta} = \frac{R}{\sin 90}$

▶ $\frac{f_1}{\cos\theta} = \frac{f_2}{\sin\theta} = R$

▶ $f_1 = R\cos\theta$ ▶ $f_2 = R\sin\theta$

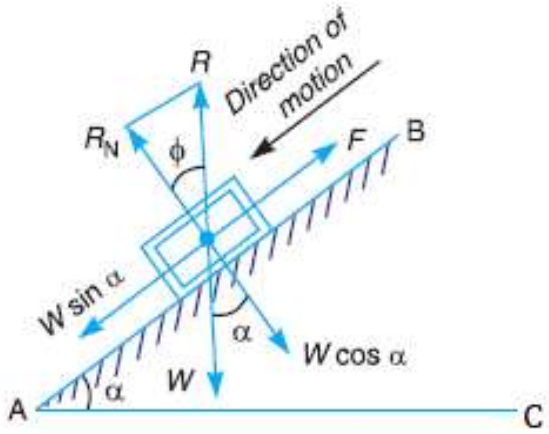


► if a body of weight (w) is placed on a smooth inclined place with the horizontal by an angle (α)

then we can resolve the weight

which acts vertically downwards

into two components



1) f_1 ⟹ in the direction of the line of the greatest slope

Where : f_1 = w sin α

1) f_2 ⟹ in the perpendicular direction on the plane

Where : f_2 = w cos α

► Resove the force of magnitude 50 newten into two components , one of them inclined on the given force an angle of measures 60° , and the other force inclined by an angle of measures 45° on the other side of the force , then approximate the answer to the nearest one decimal

SOL

► $\frac{f_1}{\sin 45} = \frac{f_2}{\sin 60} = \frac{50}{\sin 105}$

► $f_1 = \frac{50 \sin 45}{\sin 105} = 36.6$ n

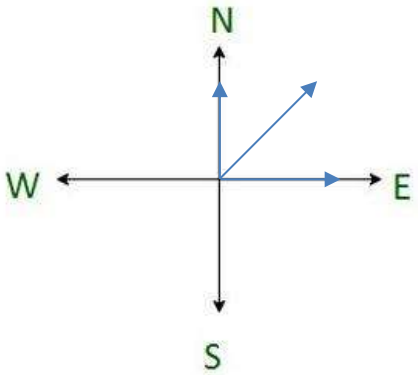
► $f_2 = \frac{50 \sin 60}{\sin 105} = 44.8$ n

► resolve the force of magnitude $12\sqrt{2}$ newton , which acts at the point O in the direction of Eastern North into two components. One of them in the East direction and the other in the North direction

SOL

► $f_1 = 12\sqrt{2} \sin 45 = 12$ n

► $f_2 = 12\sqrt{2} \cos 45 = 12$ n



► A force of magnitude $20\sqrt{3}$ n , was resolved into perpendicular components , one of them is of magnitude $10\sqrt{3}$ n .

find the magnitude of the other component

SOL

► $\frac{f_1}{\cos\theta} = \frac{f_2}{\sin\theta} = R$ ► $\frac{10\sqrt{3}}{\cos\theta} = \frac{f_2}{\sin\theta} = 20\sqrt{3}$

► $\cos\theta = \frac{10\sqrt{3}}{20\sqrt{3}}$ ► $\theta = 60^\circ$

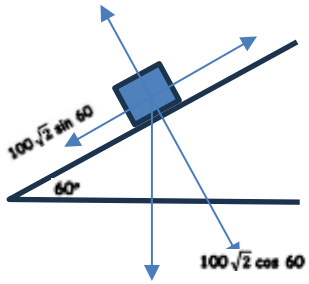
► $f_2 = 20\sqrt{3} \sin 60^\circ = 30$ n

► A bady of weight $100\sqrt{2}$ n is placed on a smooth inclined plane by 60 with the horizontal . find the two components of the weight in the direction of the line of the greatest slope and the the direction perpendicular to it

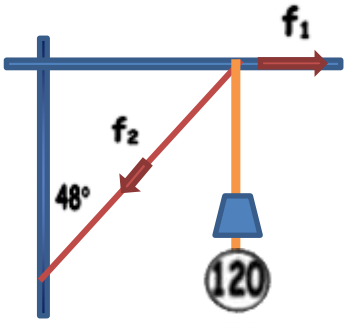
SOL

► $f_1 = 100\sqrt{2} \sin 60^\circ = 122.5$ n

► $f_2 = 100\sqrt{2} \cos 60^\circ = 70.7$ n



► in the opposite figure :



resolve the vertical force

of magnitude 120 gm.wt

into two components , one of them

in the horizontal direction and the other inclined by an angle of measure 48° with the line of action of the force

SOl

► $\frac{f_1}{\sin 48} = \frac{f_2}{\sin 90} = \frac{120}{\sin 138}$

► $f_1 = \frac{120 \sin 48}{\sin 138} = 133.3$ n

► $f_2 = \frac{120 \sin 90}{\sin 138} = 179.3$ n

► A force of magnitude 6 newten acts in direction of north .

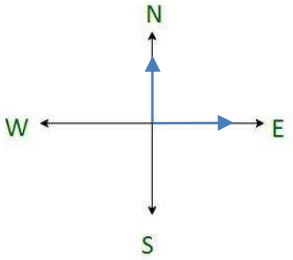
it is resolved into perpendicular components , so its component in direction of the East of magnitude ..... n

A zero B 3 C $3\sqrt{2}$ D 6

SOL

► $f_1 = 6 \cos 0 = 6$ (North)

► $f_2 = 6 \sin 0 = \text{zero}$ (East)



► A force of magnitude $4\sqrt{2}$ newten acts in direction of East .

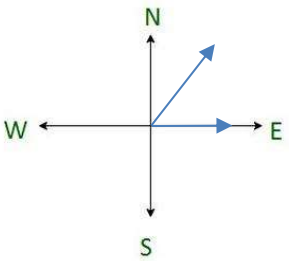
it is resolved into perpendicular components , so its component in direction of Northern East of magnitude ..... n

A zero B $4\sqrt{2}$ C 4 D 6

SOL

► $f_1 = 4\sqrt{2} \cos 45 = 4$ (Northern East)

► $f_2 = 4\sqrt{2} \sin 45 = 4$ (Southern East)



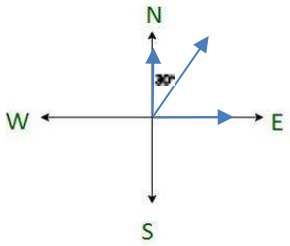
► the magnitude of a force is $5\sqrt{3}$ newten and acts in direction 30° East of the North is resolved into perpendicular components , then the magnitude of its component in the East direction = ... n

A $\frac{5\sqrt{3}}{2}$ B $\frac{15}{2}$ C $\frac{15\sqrt{3}}{2}$ D $15\sqrt{3}$

SOL

► $f_1 = 5\sqrt{3} \cos 30 = \frac{15}{2}$ (North)

► $f_2 = 5\sqrt{3} \sin 30 = \frac{5\sqrt{3}}{2}$ (East)



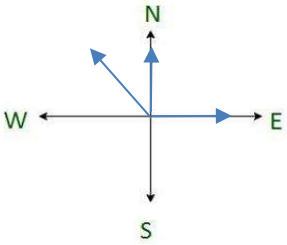
► A force of magnitude 40 newten acts vertically upwards is resolved into two components , one of them is horizontal of magnitude 20 newten , then the magnitude of the other = ... n

A 20 B $20\sqrt{3}$ C $20\sqrt{5}$ D $10\sqrt{3}$

SOL

∵ R ⊥ f_1

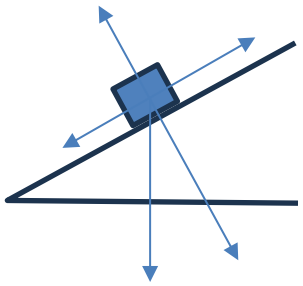
∴ $f_2 = \sqrt{40^2 - 20^2} = 20\sqrt{3}$



► if a body of weight (W) is placed on a smooth plane inclind to horizontal by angle (θ) , so the component of its weight in direction of the plane equals .....

A W B W sin θ C W cos θ D W tan θ

SOL



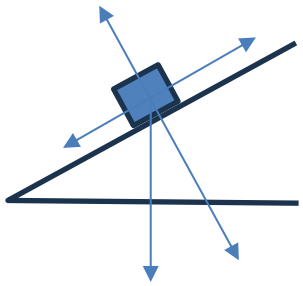
F_1 = W sin θ (in direction of the plane)

F_2 = W cos θ (in the peroendicular direction)

► a body of weight (W) placed on an inclind plane make angle (θ) With the horizontal , then the component of its weight in direction line of greatest slope and its perpendicular are 7 , 24 newten then the magnitude of the weight (W) = ... n

A 7 B 24 C 25 D 31

SOL



$\because f_1 \perp f_2$

$\therefore R = \sqrt{7^2 - 24^2} = 25$ n

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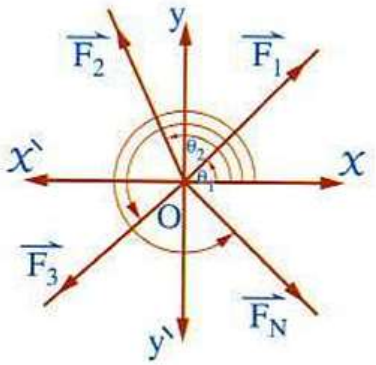
the resultant of complanar forces meeting at a point

• the resultant : $\vec{R} = X\ \vec{i} + Y\ \vec{j}$

where ...



► $X = \left(\sum_{r=1}^{n} f_r \cos\theta_r \right)$

► $Y = \left(\sum_{r=1}^{n} f_r \sin\theta_r \right)$

⟹ and the magnitude of $\vec{R}$ is $R = \sqrt{x^2 + y^2}$

and : $\tan\alpha = \frac{y}{x}$ where : $\vec{R} = (R\ ,\ \alpha)$

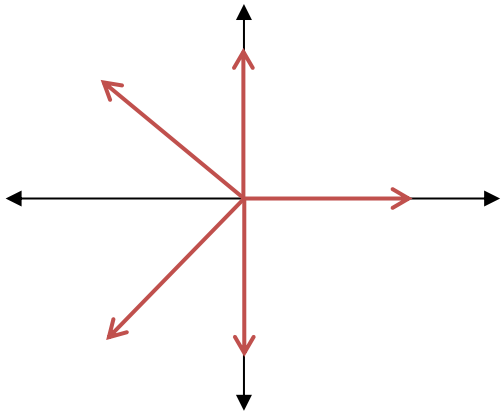
► five coplanar forces meeting at a point , their magnitude are

9 , 6 , 4 $\sqrt{2}$, 5 $\sqrt{2}$ and 5 kg.wt . in the directions :

East , North , western North , Western South and South

Prove that the set of these forces are in equilibrium

SOL



X = 9 cos 0 + 6 cos 90

+ 4 $\sqrt{2}$ cos 135 + 5 $\sqrt{2}$ cos 225

+ 6 cos 270 = zero

y = 9 sin 0 + 6 sin 90

+ 4 $\sqrt{2}$ sin 135 + 5 $\sqrt{2}$ sin 225

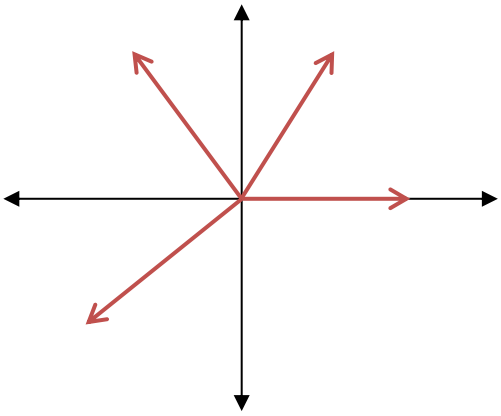
+ 6 sin 270 = zero

∵ R = zero

∴ the forces are in equilibrium

► four coplanar forces act on a particle the first of magnitude 4 n , acts in the Eastern direction, the second of magnitude 2 n , acts in direction 60 North of the East , the third of magnitude 5 n , acts in direction 60 North of West ,and the fourth of magnitude 3 $\sqrt{3}$ n , acts in direction 60 West of the South . Find the magnitude and direction of their resultant

SOL



$X = 9 \cos 0 + 6 \cos 90$

$+ 4\sqrt{2} \cos 135 + 5\sqrt{2} \cos 225$

$+ 6 \cos 270 = \text{zero}$

$y = 9 \sin 0 + 6 \sin 90$

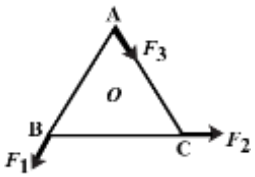
$+ 4\sqrt{2} \sin 135 + 5\sqrt{2} \sin 225$

$+ 6 \sin 270 = \text{zero}$

$\because R = \text{zero}$

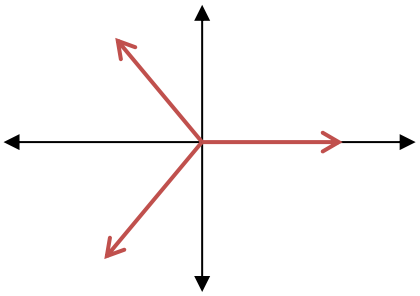
► the forces of magnitudes , 2F , 3F and 4F newton , act on a particle in the directions parallel to the sides of an equilateral triangle in the same cyclic order . Find the magnitude and the direction of the resultant of these forces

SOL



$X = 2f \cos 0 + 3f \cos 120 + 4f \cos 240$

$= 2f - \frac{3}{2} f - 2f = -\frac{3}{2} f$



$Y = 2f \sin 0 + 3f \sin 120 + 4f \sin 240$

$= 0 + \frac{3\sqrt{3}}{2} f - 2\sqrt{3} f = -\frac{\sqrt{3}}{2} f$

$\therefore R = \sqrt{(\frac{3}{2})^2 + (\frac{\sqrt{3}}{2})^2} = \sqrt{3} f$ n

$\because \tan \alpha = \frac{\sqrt{3}}{3} \quad \therefore \alpha = 180 + 30 = 210^\circ$

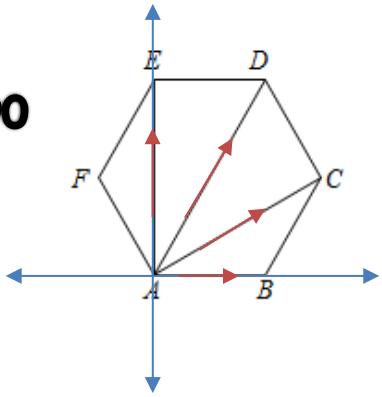
$\therefore \vec{R} = (\sqrt{3} f , 210^\circ)$

► ABCDEF is a regular hexagon , the forces of magnitudes : 8 , $6\sqrt{3}$, 5 , $4\sqrt{3}$ newton , act on $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ and $\overrightarrow{AE}$ respectively . Find the magnitude and the direction of their resultant

SOL

$X = 8 \cos 0 + 6\sqrt{3} \cos 30 + 5 \cos 60 + 4\sqrt{3} \cos 90$

$= \frac{39}{2}$



$Y = 8 \sin 0 + 6\sqrt{3} \sin 30 + 5 \sin 60 + 4\sqrt{3} \sin 90$

$= \frac{19\sqrt{3}}{2}$

$\therefore R = \sqrt{(\frac{39}{2})^2 + (\frac{19\sqrt{3}}{2})^2} = \sqrt{651}$ n

$\because \tan \alpha = \frac{19\sqrt{3}}{39}$ $\therefore \alpha = 40^\circ 9`$

► ABCD is a square of side length 12 cm , E ∈ $\overline{BC}$ where :

BE = 5 cm . forces of magnitudes 2 , 13 , $4\sqrt{2}$, 9 gm.wt newton , act on $\overrightarrow{AB}$, $\overrightarrow{CA}$, $\overrightarrow{AD}$ and $\overrightarrow{AE}$ respectively . Find the magnitude and the direction of their resultant

SOL

$X = 2\cos 0 + 13\cos\alpha - 4\sqrt{2}\cos 45 + 9\cos 90$

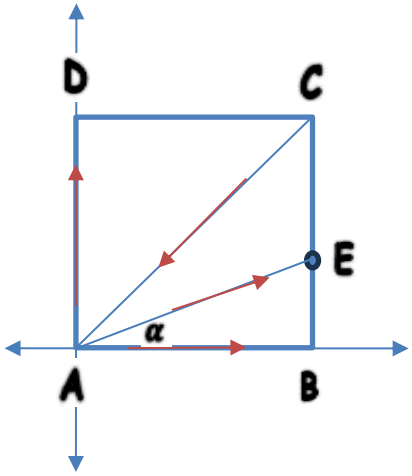
$\frac{12}{13}$

$= 10$

$Y = 2\sin 0 + 13\sin\alpha - 4\sqrt{2}\sin 45 + 9\sin 90$

$\frac{5}{13}$

$= 10$



$\therefore R = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2}$ n

$\because \tan\alpha = \frac{18}{18} = 1$ $\therefore \alpha = 45^\circ$ in the direction of $\overrightarrow{AC}$

► ABCD is a rectangle in which AB = 8 cm , BC = 6 cm , E ∈ $\overline{CD}$ where : ED = 6 cm . forces of magnitudes 12 , 40 , 26 $\sqrt{2}$ and 4 newton , act at $\overrightarrow{AB}$, $\overrightarrow{CA}$, $\overrightarrow{AE}$ and $\overrightarrow{AD}$ respectively

Find the magnitude and the direction of their resultant

SOL

$X = 12 \cos 0 - 40 \cos \alpha + 26\sqrt{2} \cos 45 + 4 \cos 90$

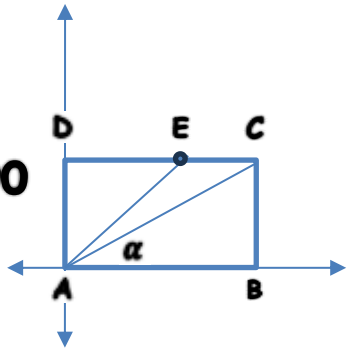
$\frac{8}{10}$

$= 6$

$Y = 12 \sin 0 - 40 \sin \alpha + 26\sqrt{2} \sin 45 + 4 \sin 90$

$\frac{6}{10}$

$= 6$



$\therefore R = \sqrt{(6)^2 + (6)^2} = 6\sqrt{2}$ n

$\because \tan \alpha = \frac{6}{6} = 1$ $\therefore \alpha = 45°$ in the direction of $\overrightarrow{AE}$

▶ if : $\vec{f_1} = 5\,\vec{i} + 3\,\vec{j}$, $\vec{f_2} = a\,\vec{i} + 6\,\vec{j}$, $\vec{f_3} = -14\,\vec{i} + b\,\vec{j}$ are three coplanar forces meeting at a point and their resultant is $\vec{R} = (10\sqrt{2}\ ,\ 135°)$. find the values of a , b

SOL

$\vec{R} = (10\sqrt{2}\cos 135°\ ,\ 10\sqrt{2}\sin 135°) = (-10\ ,\ 10)$

$\therefore\ 5 + a - 14 = -10 \quad \therefore\ a = -1$

$\therefore\ 3 + 6 + b = 10 \quad \therefore\ b = 1$

$\Longrightarrow$ if : $\vec{f_1} = 3\vec{I} - 2\vec{J}$, $\vec{f_2} = a\vec{I} - \vec{J}$, $\vec{f_3} = 4\vec{I} - b\vec{J}$, $\vec{R} = 6\vec{I} - 4\vec{J}$, then : (a , b) = ...

A (1 , -1) B (-1 , 1) C (-1 , -1) D (1 , 1)

SOL

▶ $3 + a + 4 = 6 \quad a = -1$

▶ $-2 - 1 - b = -4 \quad b = 1$

$\therefore$ (a , b) = (1 , -1)

$\Longrightarrow$ if : $\overrightarrow{f_1} = 3\,\overrightarrow{I} + 2\,\overrightarrow{J}$, $\overrightarrow{f_2} = a\,\overrightarrow{I} + 7\,\overrightarrow{J}$, $\overrightarrow{f_3} = -12\,\overrightarrow{I} + b\,\overrightarrow{J}$

are three coplanar forces metting at a point and the resultant

$\overrightarrow{R} = (6\sqrt{2} , \frac{3}{4}\pi)$, then : a - b = ...

A -3 B 3 C zero D 6

SOL

$\overrightarrow{R} = (6\sqrt{2} \cos 135° , 6\sqrt{2} \sin 135°) = (-6 , 6)$

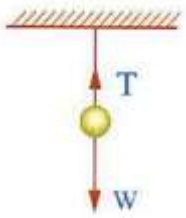
▶ $3 + a - 12 = -6$ $\therefore$ $a = 3$

▶ $2 + 7 + b = 6$ $\therefore$ $b = -3$

then : a - b = zero

Equilibrium of a rigid body under the effect of two forces / three forces meeting at a point

⟹ the rigid body is in equilibrium under the action of two forces , if the two forces :



2) Are opposite in direction

3) Their lines of action are on the same straight line

⟹ if three forces acting at a point are in equilibrium , then : the resultant of any two forces of them is equal in magnitude to the third force and acts in the opposite direction of it , and they have the same line of action

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Rule (1)

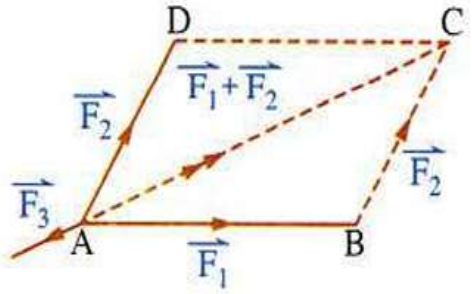
if three forces are acting at a point

and can be represented

by the sides of a triangle

taken in the same cyclic order

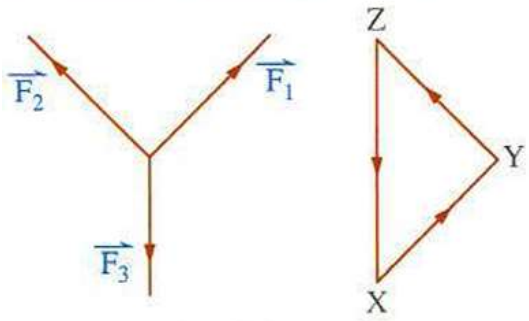
then the forces are in equilibrium



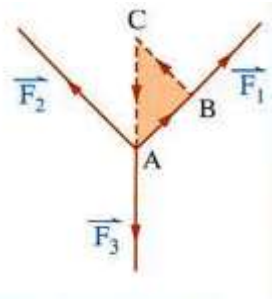
Rule (2) < the triangle of forces >

if a rigid body is in equilibrium under the action of three forces acting at a point and a triangle is drawn whose sides are parallel to the lines of action of the forces and taken in the same cyclic order , then the lengths of the sides of the triangle are proportional to the magnitude of the corresponding forces .

$$\Longrightarrow \frac{f_1}{XY} = \frac{f_2}{YZ} = \frac{f_3}{XZ}$$

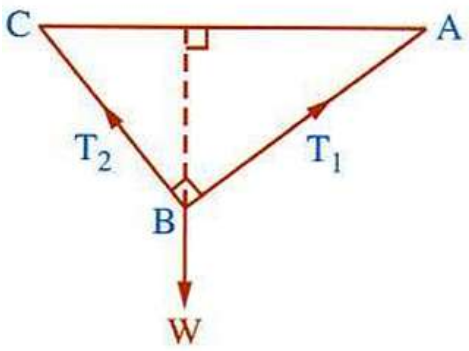


Remark ...



such that two of its sides are on the line

is parallel to the line of action of the third force



if a triangle is drawn in which

its sides are perpendicular

to the directions of the equilibrium

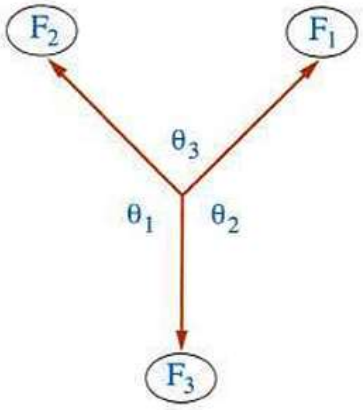
then the ratio between each force and the

length of the side p erpendicular to it is equal

$$\Longrightarrow \frac{T_1}{AB} = \frac{T_2}{BC} = \frac{W}{AC}$$

Rule (3) < Lami's rule >

if three coplanar forces meeting at a point and acting up on a particle are in equilibrium , then the magnitude of each force is proportional to the sine angle between the two other forces .



$$\Longrightarrow \frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

► three coplanar forces of magnitudes 60 , F , K newton meeting at a point they are represented by the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ of Δ ABC respectively where : AB = 3 cm , BC = 4 cm , AC = 5 cm find the value of each of f_1 and f_2

SOL

$$\because \frac{F_1}{AB} = \frac{F_2}{BC} = \frac{F_3}{AC} \qquad \therefore \frac{60}{3} = \frac{F}{4} = \frac{K}{5}$$

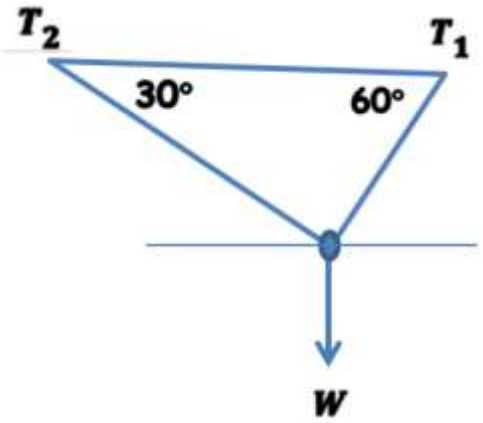
► $f = \frac{4 \times 60}{3} = 80$ n

► $k = \frac{5 \times 60}{3} = 100$ n

► A weight of magnitude 25 n is suspended by two strings , the first is inclined by an angle of measure 60° to the horizontal and the second is inclined by an angle of measure 30° to the horizontal . find : T_1 , T_2 in case of equilibrium

SOL

$$\because \frac{W}{\sin\theta_1} = \frac{T_1}{\sin\theta_2} = \frac{T_2}{\sin\theta_3}$$



$$\therefore \frac{25}{\sin 90} = \frac{T_1}{\sin 120} = \frac{T_2}{\sin 150}$$

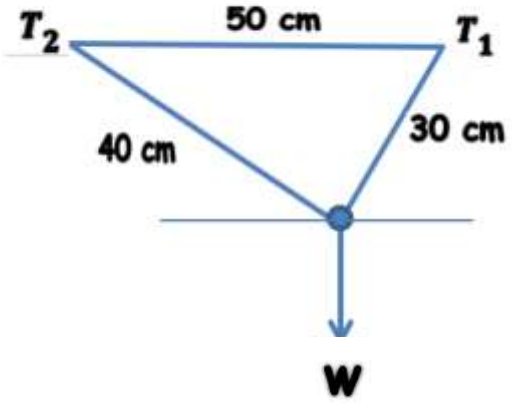
$$\therefore T_1 = \frac{25 \sin 120}{\sin 90} = \frac{25}{2}\sqrt{3}\ n$$

$$, T_2 = \frac{25 \sin 150}{\sin 90} = \frac{25}{2}\ n$$

► A weight of magnitude 150 gm.wt is suspended by two strings of lengths 30 cm and 40 cm , from two points on one horizontal line where the distance between them is 50 cm . find : the magnitude of tension in e ach of the two strings in case of equilibrium

SOL

$\because \frac{W}{L_1} = \frac{T_1}{L_2} = \frac{T_2}{L_3}$



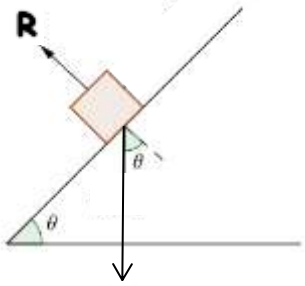
$\therefore \frac{150}{50} = \frac{T_1}{30} = \frac{T_2}{40}$ W

$\therefore T_1 = \frac{30 \times 150}{50} = 90$ gm. wt

, $T_2 = \frac{40 \times 150}{50} = 120$ gm.wt

• **Equilibrium of a body placed on a smooth inclined plane :**

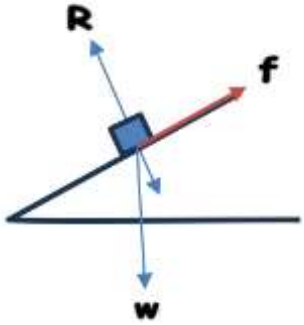
if a body of weight (w) is placed on a smooth inclined plane which inclines by an angle of measure θ with the horizontal then the body will be under the action of two forces :



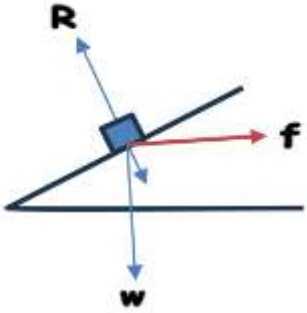
► the weight force (w) acting vertically downwards

► the reaction force (R) of the inclined plane acting in direction perpendicular to the plane

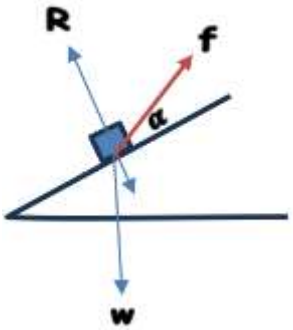
⟹ these two forces cannot be equilibrium because they have two different lines of action . therefore we need a third force act on the body to be in equilibrium .



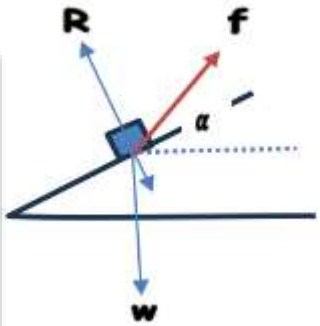
the force in the direction of the line of the greatest slope upwards



the force act horizontally



the force inclines by α with the plane upwards

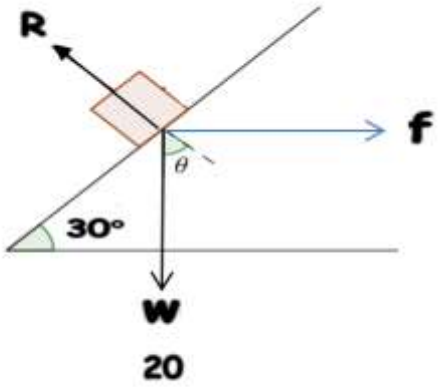


the force inclines by α with the horizontal upwards

► A body of weight 20 kg.wt. is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , it is kept in equilibrium by a horizontal force . find the magnitude of the weight W and the magnitude of the reaction of the plane

SOL

$$\because \quad \frac{W}{\sin \theta_1} = \frac{R}{\sin \theta_2} = \frac{F}{\sin \theta_3}$$



$$\therefore \quad \frac{20}{\sin 120^\circ} = \frac{R}{\sin 90^\circ} = \frac{F}{\sin 150^\circ}$$

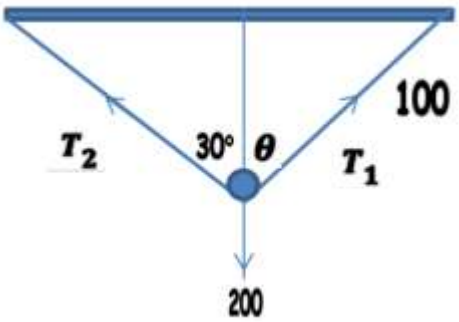
$$\therefore \ R = \frac{20 \times \sin 90^\circ}{\sin 120^\circ} = \frac{40}{3}\sqrt{3} \text{ gm.wt}$$

$$\therefore \ F = \frac{20 \times \sin 150^\circ}{\sin 120^\circ} = \frac{20}{3}\sqrt{3} \text{ gm.wt}$$

► A particle of weight 200 gm.wt. is suspended by two strings . one of them is inclined to the vertical by angle of measure θ and the other inclined to the vertical by an angle of measure 30° . if the magnitude of the tension in the first string is 100 gm.wt. find θ and the magnitude of the tension in the second string

SOL

$$\because \frac{W}{\sin\theta_1} = \frac{T_1}{\sin\theta_2} = \frac{T_2}{\sin\theta_3}$$



$$\therefore \frac{200}{\sin(\theta+30°)} = \frac{100}{\sin 150°} = \frac{T_2}{\sin(180°-\theta)}$$

$$\therefore \frac{200}{\sin(\theta+30°)} = \frac{100}{\sin 150°} = \frac{T_2}{\sin\theta}$$

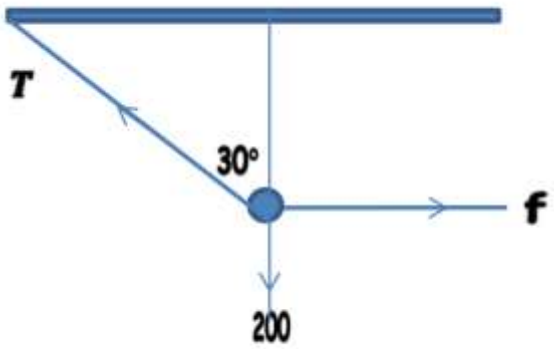
$$\therefore \sin(\theta+30) = \frac{200\sin 150°}{100} = 1 \qquad \therefore \theta = 60°$$

$$\therefore T_2 = \frac{100\sin 60°}{\sin 150°} = 100\sqrt{3} \text{ gm.wt.}$$

► A weight of weight 200 gm.wt. is suspended at the end of a light string ,the other end of which is attached to the ceiling of a room . the weight is pulled by a horizontal force until the string is inclined to the vertical by an angle of measure 30° . find the magnitude of each of the horizontal force and the tension in the string

SOL

$\because \quad \frac{W}{\sin \theta_1} = \frac{T}{\sin \theta_2} = \frac{F}{\sin \theta_3}$



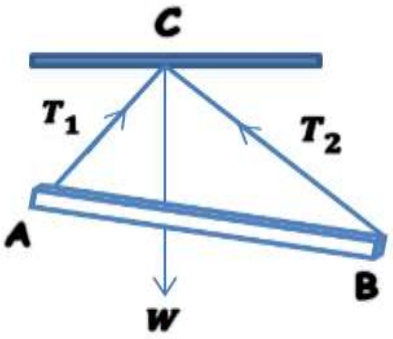
$\therefore \quad \frac{200}{\sin 120°} = \frac{T}{\sin 90°} = \frac{F}{\sin 150°}$

$\therefore \quad T = \frac{200 \sin 90°}{\sin 120°} = 231 \text{ gm.wt.}$

$\therefore \quad F = \frac{200 \sin 150°}{\sin 120°} = 115 \text{ gm.wt.}$

► $\overline{AB}$ is A uniform rod of length 100 cm , and weight 30 kg.wt. is suspended from its two ends A and B by means of two strings , their other ends are fixed at a point in the ceiling at the point C , if the two strings are perpendicular and AC = 50 cm . find the tension in each of the two strings

SOL



$\because \frac{W}{AB} = \frac{T_1}{AC} = \frac{T_2}{BC}$

$\therefore \frac{30}{100} = \frac{T_1}{50} = \frac{T_2}{50\sqrt{3}}$

$\therefore T_1 = \frac{200 \sin 90°}{\sin 120°} = 231$ km.wt.

$\therefore T_2 = \frac{200 \sin 150°}{\sin 120°} = 115$ km.wt.

⟹ the least number of coplanar unequal in magnitude forces could be in equilibrium is .....

A 1 B 2 C 3 D 4

SOL

► the least number is 3

⟹ if : f is in equilibrium with two perpendicular forces of magnitudes 8 newten and 15 newten , then : f = ... newten

A 7 B 17 C 23 D $7\sqrt{2}$

SOL

► $f = \sqrt{8^2 + 15^2} = 17$ newten

⟹ if : f is in equilibrium with two forces of magnitudes 5 newten and 3 newten and the measure of the angle between them is 60° then : f = ... newten

A $\sqrt{19}$ B $\sqrt{34}$ C 7 D 15

SOL

► $f = \sqrt{5^2 + 3^2 + 2(5)(3)\cos 60} = 7$ newten

⟹ three coplanar forces are in equilibrium act at a particle , the measure of the angle between the first two forces is 60° , and between the second and third forces is 150° . then the ratio between forces is .....

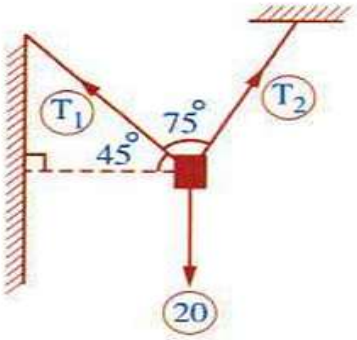
A 1 : 1 : $\sqrt{3}$ B 1 : 2 : $\sqrt{3}$ C $\sqrt{2} : \sqrt{3} : 1$ D $\sqrt{3} : \sqrt{3} : 1$

SOL

► $\frac{f_1}{\sin 150} = \frac{f_2}{\sin 150} = \frac{f_3}{\sin 60}$

► $f_1 : f_2 : f_3 = \frac{1}{2} : \frac{1}{2} : \frac{\sqrt{3}}{2} = 1 : 1 : \sqrt{3}$

⇒ in the opposite figure :



The weight of a body is 20 kg.wt

The body is in equilibrium

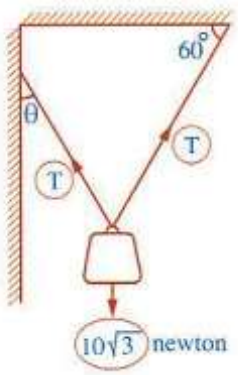
then : $\frac{t_1}{t_2}$ = ...

A $\frac{1}{2}$ B $\frac{1}{\sqrt{2}}$ C $\frac{3}{2}$ D $\frac{\sqrt{3}}{2}$

SOL

▶ $\frac{20}{\sin 75} = \frac{t_1}{\sin 150} = \frac{t_2}{\sin 135}$ ▶ $\frac{t_1}{t_2} = \frac{\sin 135}{\sin 150} = \frac{1}{\sqrt{2}}$

⇒ in the opposite figure :



a body of weight $10\sqrt{3}$ newten is suspended by two strings as shown in the figure , then the value of θ which makes both tensions are equal is .....

A 15° B 30° C 45° D 60°

SOL

▶ θ = 30°

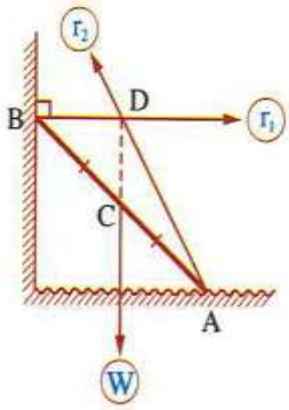
Follow : Equilibrium of a rigid body under the effect of three forces meeting at a point

Rule (4)

If a rigid body is an equilibrium under the action of three coplanar forces , then the lines of action of forces meet at a point

→ if a uniform rod of wight (W) is in equilibrium

On a smooth vertical wall and on rough ground



Then :

1 the weight of the rod acts vertically downwards at its midpoint (centre of gravity)

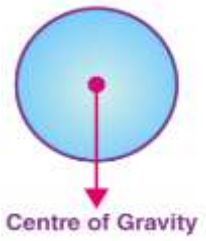
2 the reaction of the smooth vertically wall (r_1) which is perpendicular to the wall

3 the reaction of the rough ground (r_2) with unknown direction , and to determine its direction , we draw a ray form the meeting point with the ground and passes through to the point of intersection between $\vec{W}$, $\vec{r_1}$

Remarks ...

1 the weight of the uniform sphere acts

at its geometric centre (centre of gravity)



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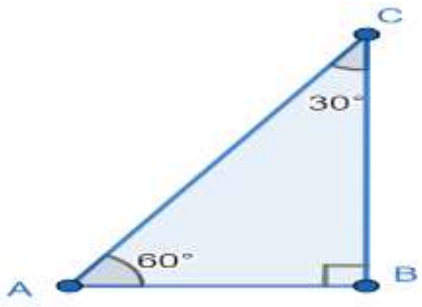
2 if Δ ABC is 30° - 60° triangle

Then : $AB = \frac{1}{2} AC$, $BC = \frac{\sqrt{3}}{2} AC$



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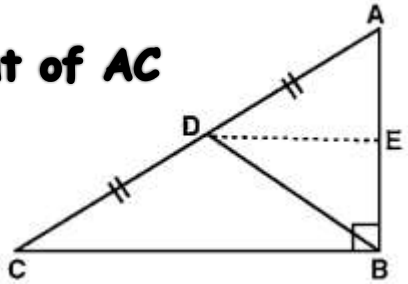
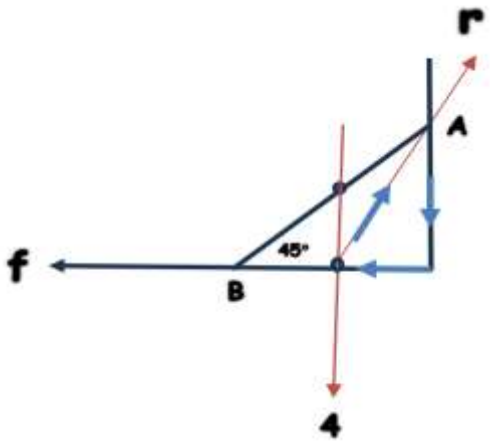
3 if Δ ABC is right-triangle at B , D a midpoint of AC

Then : $BD = \frac{1}{2} AC$, $DE = \frac{1}{2} BC$



$\Longrightarrow \overline{AB}$ is a uniform rod , its end A is attached by a hings fixed in a vertical wall a horizontal force act at the end B to keep the rod in equilibrium while it is inclined to the wall by an angle of measure 45° , if the weight of the rod is 4 kg.wt . act at its midpoint , then find the magnitude of the force and the reaction of the hinge

SOL

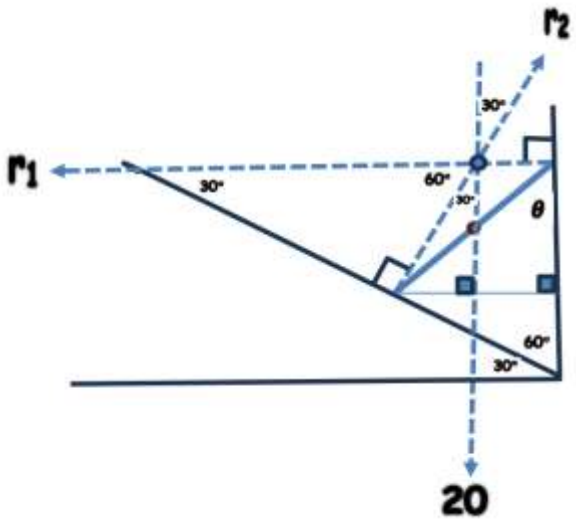


▶ $\frac{4}{1} = \frac{f}{\frac{1}{2}} = \frac{r}{\frac{\sqrt{5}}{2}}$

▶ $r = 2\sqrt{5}$ kg.wt

⟹ A uniform rod of weight 20 kg.wt. rests at one of its ends against a smooth vertical plane and at the other end on a smooth plane inclined to the vertical at an angle of measure 60° , in the state of equilibrium . find the magnitude of each of the two reactions of the two planes , also find the angle at which the rod inclines to the vertical

SOL



▶ $\frac{20}{\sin 120} = \frac{r_1}{\sin 150} = \frac{r_2}{\sin 90}$

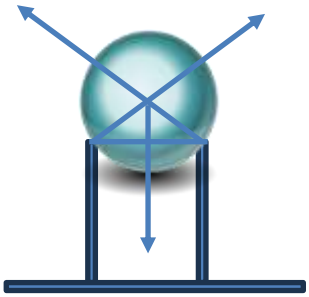
▶ $r_1 = \frac{20 \sin 150}{\sin 120} = \frac{20\sqrt{3}}{3}$ kg.wt.

▶ $r_2 = \frac{20 \sin 90}{\sin 120} = \frac{40\sqrt{3}}{3}$ kg.wt.

▶ $\tan \theta = \frac{2}{\sqrt{3}}$ ▶ $\theta \simeq 49°$

⟹ A homogeneous sphere rests on two parallel rods lie on the same horizontal plane the distance between them equals the radius length of the sphere . find the pressure on each rod if the weight of the sphere is 60 newton in case of equilibrium

SOL



▶ $\frac{60}{\sin 60} = \frac{r_1}{\sin 150} = \frac{r_2}{\sin 150}$

▶ $r_1 = r_2 = \frac{60 \sin 150}{\sin 60} = 20\sqrt{3}$ newton

---

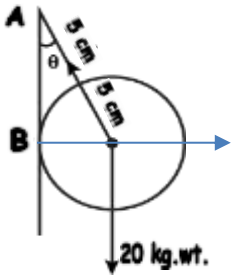
⟹ A solid uniform sphere of weight 20 kg.wt. and its radius length 5 cm . if it is in equilibrium by a string of length 5 cm . attached to a point of its surface and the other end of the string is fixed at a point in the vertical smooth plane above the tangency point , then the reaction of the vertical plane r = …… kg.wt.

A $\frac{20}{\sqrt{3}}$ B 20 C $\frac{20}{\sqrt{5}}$ D zero

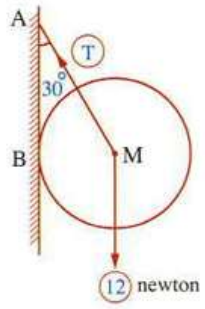
SOL

$AB = \sqrt{10^2 - 5^2} = 5\sqrt{3}$

▶ $\frac{20}{5\sqrt{3}} = \frac{r}{5}$ ▶ $r = \frac{20}{\sqrt{3}}$



⟹ **in the opposite figure :**



if the sphere is in equilibrium

Then : T - r = ... newton

A $8\sqrt{3}$ B $4\sqrt{3}$ C 4 D 8

SOL

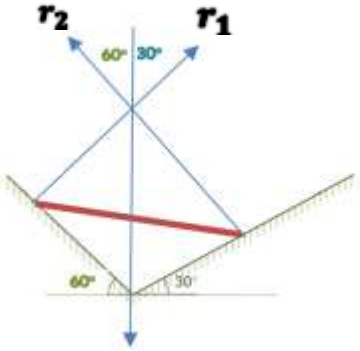
▶ $\frac{12}{\sin 120} = \frac{r}{\sin 150} = \frac{t}{\sin 90}$

▶ $t - r = \frac{12 \sin 90}{\sin 120} - \frac{12 \sin 150}{\sin 120} = 4\sqrt{3}$

⟹ A uniform rod of weight 24 newten is placed on two smooth planes inclined at two angles of measures 60° and 30° to the horizontal , then the magnitude of the pressure on each plane ..... newten

A 12 , 15 B 12 , $12\sqrt{3}$ C $12\sqrt{3}$, 10 D 15 , 13

SOL



▶ $\frac{24}{\sin 90} = \frac{r_1}{\sin 120} = \frac{r_2}{\sin 150}$

▶ $r_1 = 12\sqrt{3}$, $r_2 = 12$